SPEAKERS, TITLES, AND ABSTRACTS FOR JAF 2019 AT THE CUNY GRADUATE CENTER NEW YORK CITY

Speaker: Athar Abdul-Quader (SUNY Purchase College)

Title: Neutrality

Abstract: A subset of a model of PA is called neutral if it does not change the definable closure relation. A model PA with undefinable neutral classes is called neutrally expandable. Clearly, every 0-definable set is neutral in any model. We give examples of neutrally expandable models and prove that recursively saturated models are not neutrally expandable. We also study a local version of neutrality: sets which, for a given subset A of a model, do not change the definable closure relation for elements of A.

Speaker: Yong Cheng (Wuhan University)

Title: Essential Undecidability of Weak Arithmetics

Abstract: In this talk, I examine the limit of incompleteness for weak arithmetics w.r.t. interpretation. I first define the notion "Gödel's first incompleteness theorem ($\mathsf{G1}$ for short) holds for theory T". This work is motivated by the following question: whether there is a weaker theory T than Robinson's $\mathbf{R}$ w.r.t. interpretation such that $\mathsf{G1}$ holds for T? In this work, I show that there are many such theories via two different methods. The first proof is based on Jeřábek's work using some model theory and I show that from each recursively inseparable pair we can construct a weaker theory than $\mathbf{R}$ w.r.t. interpretation such that $\mathsf{G1}$ holds for it. The second proof is based on Shoenfield's work using some recursion theory and I show that for any Turing degree $\mathbf{0} < \mathbf{d} < \mathbf{0}'$, there is a weaker theory than $\mathbf{R}$ w.r.t. interpretation with Turing degree $\mathbf{d}$ such that $\mathsf{G1}$ holds for it. As two corollaries, I answer a question from Albert Visser and show that there is no weakest theory below $\mathbf{R}$ w.r.t. Turing degrees such that $\mathsf{G1}$ holds for it.

Speaker: Gabriel Conant (Notre Dame University)

Title: Tame Expansions of the Group of Integers

Abstract: In the context of model theoretic complexity, the group of integers and the ring of integers sit at opposite ends of the spectrum. The group of integers has a decidable theory and admits meaningful classification of definable sets, while the ring of integers is undecidable and its definable sets are highly complicated. This talk will be about intermediate structures between $(\mathbb{Z}, +, 0)$ and $(\mathbb{Z}, +, \cdot, 0, 1)$, and the transition from "tame" to "wild". The main focus will be on combinatorial and number theoretic conditions on subsets $A \subseteq \mathbb{Z}$ which imply that the expansion of $(\mathbb{Z}, +, 0, A)$ remains well-behaved with respect to model theoretic notions of tameness in definable sets.

Speaker: Damir Dzhafarov (University of Connecticut)

Title: The first-order parts of Weihrauch degrees

Abstract: The Weihrauch degrees represent degrees of unsolvability of various mathematical problems. Their study has been widely applied in computable analysis, complexity theory, and more recently, also computable combinatorics. For problems expressible as Π^1_2 principles of second-order arithmeitc, the Weihrauch analysis largely extends the traditional framework of reverse mathematics, providing a more refined picture of a problem's second-order strength. We introduce the concept of the first-order part of a Weihrauch degree that seeks to similarly refine a problem's first-order strength. We classify, and obtain bounds on, the first-order part of many natural problems, and show that this gives rise to a hierarchy of principles that roughly corresponds to the Kirby-Paris hierarchy of first-order arithmetic. This is joint work with Reed Solomon and Keita Yokoyama.

Speaker Victoria Gitman (CUNY)

Title: A model of second-order arithmetic satisfying AC but not DC

Abstract: Models of second-order arithmetic have two kinds of objects: numbers and sets of numbers, which we think of as the reals. A second-order arithmetic axiom system specifies general existence rules for real numbers. If a result from, say, analysis can be proved in some such system, then we have a bound on the kind of reals that must exist in order for it to hold. Indeed, most classical results in analysis don't just follows from, but are actually equivalent (over a weak base system) to one of a short list of main second-order arithmetic systems.

One of the strongest second-order arithmetic systems is *full second-order arithmetic* Z_2 which asserts that every second-order formula (with any number of set quantifiers) defines a set. We can augment Z_2 with choice principles such as the choice scheme and the dependent choice scheme. The Σ^1_n-*choice scheme* asserts for every Σ^1_n-formula $\varphi(n, X)$ that if for every n, there is a set X witnessing $\varphi(n, X)$, then there is a single set Z whose n-th slice Z_n is a witness for $\varphi(n, X)$. The Σ^1_n-*dependent choice scheme* asserts that every Σ^1_n-relation $\varphi(X, Y)$ without terminal nodes has an infinite branch: there is a set Z such that $\varphi(Z_n, Z_{n+1})$ holds for all n. The system Z_2 proves the Σ^1_2-choice scheme and the Σ^1_2-dependent choice scheme. The independence of Π^1_2-choice scheme from Z_2 follows by taking a model of Z_2 whose sets are the reals of the Feferman-Levy model of ZF in which every $\aleph^L_n$ is countable and $\aleph^L_\omega$ is the first uncountable cardinal.

We construct a model of ZF + AC_ω whose reals give a model of Z_2 together with the full choice scheme in which Π^1_2-dependent choice fails. This result was first proved by Kanovei in 1979 and published in Russian. It was rediscovered by Sy Friedman and myself with a slightly simplified proof.

Speaker: Michał Tomasz Godziszewski (University of Warsaw)

Title: Π^0_1-computable quotient presentations of nonstandard models of arithmetic

Abstract: A computable quotient presentation of a mathematical structure $\mathcal{A}$ consists of a computable structure on the natural numbers $\langle \mathbb{N}, \star, *, \dots \rangle$, meaning that the operations and relations of the structure are computable, and an equivalence relation E on $\mathbb{N}$, not necessarily computable but which is a congruence with respect to this structure, such that the quotient $\langle \mathbb{N}, \star, *, \dots \rangle$ is isomorphic to the given structure $\mathcal{A}$. Thus, one may consider computable quotient presentations of graphs, groups, orders, rings and so on. A natural question asked by B. Khoussainov in 2016, is if the Tennenbaum Thoerem extends to the context of computable presentations of nonstandard models of arithmetic. In a joint work with J.D. Hamkins we have proved that no nonstandard model of arithmetic admits a computable quotient presentation by a computably enumerable equivalence relation on the natural numbers. However, as it happens, there exists a nonstandard model of arithmetic admitting a computable quotient presentation by a co-c.e. equivalence relation. Actually, there are infinitely many of those. The idea of the proof consists in simulating the Henkin construction via finite injury priority argument. What is quite surprising, the construction works (i.e. injury lemma holds) by Hilbert's Basis Theorem. During the talk I'll present ideas of the proof of the latter result, which is joint work with T. Slaman and L. Harington.

Speaker: Simon Heller (The CUNY Graduate Center)

Title: Modest automorphisms of Presburger arithmetic

Abstract: This talk will summarize some results, such as quantifier elimination and DP rank, for an expansion of Presburger arithmetic by a single automorphism, and will sketch proofs of those results, as well as some areas of possible future research.

Speaker: Matt Kaufmann (University of Texas at Austin)

Title: Logical Foundations for the ACL2 Theorem Prover

Abstract: ACL2 ("A Computational Logic for Applicative Common Lisp") is a programming language, a first-order logic, and a software system used in industry for proving theorems in that logic. I'll start by giving an introduction to ACL2. Then I'll discuss foundational issues for ACL2.

Speaker: Whan Ki Lee (Queensborough Communty College)

Title: Resplendent models generated by indiscernibles.

Abstract: Recursively saturated structures are considered as being large because they realize all of their recursive types, and structures generated by indiscernibles as being small because all their elements are definable from a set of indiscernibles. However, it turns out that the two classes of structures are closely related. We will discuss the relation between the two classes in weak fragments of arithmetic.?

Speaker: Anders Lundstedt (Stockholm University)

Title: Necessarily non-analytic induction proofs

Abstract: Sometimes when trying to prove a fact F by induction one gets "stuck" when trying to prove the induction step. The solution is sometimes to instead prove a "stronger" fact S by induction. This proof method is usually called something like "strengthening of the induction hypothesis". However, there need not always be a precise sense in which the fact S is "stronger". Thus, following Hetzl and Wong (2018), we use the more general terminology "non-analytic induction proofs" for such proofs. A natural question for such proofs is whether the non-analyticity is necessary—that is, whether one could prove F without the "detour" via proving S. Hetzl and Wong have made precise sense of this question for first-order theories and sentences of arithmetic. Based on this, we investigate whether some particular induction proofs are necessarily non-analytic.

Reference: Hetzl, Stefan and Tin Lok Wong (2018). "Some observations on the logical foundations of inductive theorem proving". *Logical Methods in Computer Science* 13.4, pp. 1–26.

Speaker: Chris Miller (The Ohio State University)

Title: The intrusion of fragments of arithmetic into definability theory of expansions of the real field.

Abstract: Let $\mathfrak{R}$ be an expansion of the real field $(\mathbb{R}, +, \cdot)$ by constructible (that is, boolean combination of open) subsets of various $\mathbb{R}^n$. Much attention has been paid to the case that $\mathfrak{R}$ is o-minimal (equivalently, defines no sets having infinitely many connected components), but there are examples of interest that are not o-minimal, in which case it is known that $\mathfrak{R}$ necessarily defines an infinite closed discrete $S \subseteq [0, \infty)$. In order to understand such $\mathfrak{R}$ we must at the very least understand the structure induced on S in $\mathfrak{R}$, that is, the expansion, $\mathfrak{S}$, of S by all subsets of each S^n that are definable in $\mathfrak{R}$. As $\mathfrak{S}$ is isomorphic to some expansion of $(\mathbb{N}, <)$, another way to put all this is that there are good reasons to consider fusions of o-minimal expansions of $(\mathbb{R}, +, \cdot)$ with isomorphs of expansions of $(\mathbb{N}, <)$. Evidently, this program is affected by the extent to which we understand expansions of $(\mathbb{R}, <, \mathbb{N})$, especially those that do not define multiplication on $\mathbb{N}$.

Speaker: Russell Miller (Queens College)

Title: Hilbert's Tenth Problem for the Rational Numbers and their Subrings.

Abstract: For a ring R, *Hilbert's Tenth Problem* is the set $HTP(R)$ of polynomials $f \in R[X_0, X_1, \ldots]$ for which $f = 0$ has a solution in R. It has been known since the work of Davis, Putnam, Robinson, and Matiyasevich in 1970 that $HTP(\mathbb{Z})$ is as hard to decide as the Halting Problem, and indeed that every computably enumerable set is *diophantine in* $\mathbb{Z}$, i.e., definable in $\mathbb{Z}$ by an existential formula, and thus 1-reducible to $HTP(\mathbb{Z})$. In contrast, the decidability of $HTP(\mathbb{Q})$ is an open question.

We approach this problem for $\mathbb{Q}$ by examining it on the subrings R of $\mathbb{Q}$. These subrings form a topological space with a natural measure, having the property of Baire. It turns out that for almost all these subrings R, there exists a set that is R-computably enumerable but not diophantine in R, and indeed not 1-reducible to R: this holds outside a meager set of measure 0 in the space. We say that these subrings fail to be *HTP-complete*. On the other hand, joint work by Ken Kramer and the speaker has shown that every Turing degree contains an HTP-complete set. Finally, we will present results relating properties of $HTP(\mathbb{Q})$ to the prevalence of those properties among the sets $HTP(R)$ for subrings R of $\mathbb{Q}$.

Speaker: Arseniy Sheydvasser (GC CUNY)

Title: Applications of Model Theory to Families of Integer Sequences

Abstract: Suppose that you have a family of integer sequences s_n, and a corresponding algorithm $P(k, n)$ that computes the first k terms of s_n. Using an ultrafilter, you can extend this family of sequences to a family indexed by hyper-naturals. The algorithm $P(k, n)$ also extends, but generally will no longer be a proper algorithm (because it may run forever). What can we say if $P(k, n)$ remains a proper algorithm even for some hyper-natural inputs? What does this imply about the underlying family of integer sequences? In this talk, we'll discuss some known examples, and show the remarkable results that can be proved, suggesting that there may be some underlying theory that we do not understand yet.

Speaker: Erez Shochat (St. Francis College)

Title: On Countable Short Recursively Saturated Models of Arithmetic and Their Automorphisms

Abstract: Countable short recursively saturated models of arithmetic are elementary initial segments of countable recursively saturated models of arithmetic. In this talk we outline the properties of these models and discuss their automorphism groups.

Speaker: Mikhail Starchak (St. Petersburg State University)

Title: Some definability results for $x(x+1) \mid y$ and elimination of quantifiers for the positive $\exists\mathrm{Th}\,\langle \mathbb{N}; 1, +, -, \perp\rangle$

Abstract: Two problems considered in the talk are independent but they are motivated by the study of time-complexity of the decision problem for $\exists\mathrm{Th}\,\langle \mathbb{N}; 1, +, |\rangle$. This theory is NP-hard and in NEXPTIME but not known to be in NP. We see that the binary length of the smallest satisfying assignment for the formula

$$\bigwedge_{i=1}^{m} (1 + x_i \mid 1 + x_{i+1} \wedge 2 + x_i \mid 1 + x_{i+1})$$

is bounded exponentially in the length of the formula. If we define the predicate $DW(x, y) \rightleftharpoons x \mid y \wedge 1 + x \mid y$, we can ask whether the theory $\exists\mathrm{Th}\,\langle \mathbb{N}; S, DW\rangle$ is in NP, or can we bound a satisfying assignment for every formula φ from $\exists\mathrm{Th}\,\langle \mathbb{N}; 1, +, \perp\rangle$ polynomially in the length of φ?

We consider some definability properties for DW. The structures $\langle \mathbb{N}; |, DW\rangle$ and $\langle \mathbb{N}; +, DW\rangle$ are def-complete. Existential definability of the successor function and equality in the structure $\langle \mathbb{N}; \cdot, DW\rangle$ implies RE-completeness of the decision problem for $\exists\mathrm{Th}\,\langle \mathbb{N}; \cdot, DW\rangle$.

Since the predicate $x \nmid y$ is existentially definable in the structure $\langle \mathbb{N}; 1, +, | \rangle$, we can not eliminate quantifiers in the *positive* existential theory $\exists \mathrm{Th} \langle \mathbb{N}; 1, +, -, | \rangle$. But this is possible if we substitute divisibility on coprimeness. The quantifier elimination procedure will be sketched in the second part of the talk.

Speaker: Corey Switzer (The CUNY Graduate Center)

Title: $(\mathcal{L}, n)$-Models

Abstract:$(\mathcal{L}, n)$-Models are sequences of finite models approximating a model of an arithmetic theory as formalized in PA. These sequences were first introduced by Shelah and recently studied by the speaker. They allow one to turn questions about consistency and completeness into finitary, combinatorial statements in a straightforward manner, thus giving a relatively uniform and systematic approach to "mathematical" independence in PA. In this talk we will outline their general theory as well as some applications. These include Shelah's example of a true but unprovable Π^0_1 statement and our recent variation.